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Children as insurance

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Abstract. This paper presents a dynamic model of fertility decisions in which children serve as an incomplete insurance good. The model incorporates uncertainty about future income and the survival of children as well as a discrete representation of the number of children. It contributes to the understanding of the negative relation between fertility and education, shows why parents may demand children even if the return is negative, and explains why fertility might rise with increasing income when income is low and decrease when income is high. Furthermore, the model can account for the decline in fertility when the risk of infant and child mortality decreases. Finally, the implications for empirical tests of the demand for children are also examined.

JEL classification: J13, O12, D11

Key words: Fertility, mortality, insurance, uncertainty

1. Introduction

> [A]n important question is whether having many children and/or a large extended household is an optimizing strategy allowing households to derive benefits otherwise lost due to poorly functioning markets ... (Birdsall 1988, p 502)

In less developed countries insurance and credit markets are examples of poorly functioning or absent markets that may affect the demand for children.

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With little possibility for risk diversification, the need for insurance has to be satisfied by other means. The hypothesis of this paper is that children, under these circumstances, can serve as an incomplete insurance good.

Most theoretical and empirical analyses of children as security assets have focused on the old-age security aspect of children.[1] Cain (1981, 1982, 1983) has, however, emphasised that children can also provide insurance against shortfalls in income under other circumstances. Similarly, Jones (1987, chapt. 1) argues that the near universal marriage pattern and higher fertility found in Asia compared with Europe in pre-modern time was not due to old-age security considerations. Instead it arose from the need to "... command as much labour as possible to help recover from the effects of recurrent disaster" (Jones 1987, p 17). Yet, formal modelling of decisions when children serve as a general substitute for insurance is lacking. Appelbaum and Katz (1991) analysed fertility decisions when the income of individual household members (including children) is uncertain. Yet, their model is static, and they do not explicitly examine the causes of uncertainty in children's income.

In this paper, I analyse fertility decisions of a household when it faces uncertainty with respect to future income and child survival. Children are costly in the first period of their life and provide a positive net income in subsequent periods. The total net income from a child over the parents' lives is assumed to be at most zero and is uncertain because of mortality. Hence, parents use children as a means to shift income from a period with certain income to future periods with uncertain income, thereby insuring themselves against the possibility of low income. Assuming realistically that the number of children can only take integer values a binomial distribution is used to model mortality. This was first suggested by Sah (1991). He used the approach to analyse the effects of mortality changes on fertility and parents' utility in a model where parents derive direct utility from the number of surviving children. The major advantages of using a binomial distribution are tractability and the realism of a discrete number of children.

The model has two novel aspects: Children are modelled as a general insurance and saving asset, and the dynamic aspects of income and fertility are explicitly examined. This leads to alternative explanations of a number of stylised facts about fertility. They include the strong negative correlation between mothers' education and fertility, the positive relation between income and fertility at low levels of development, and the negative relation at higher levels of development. The model also demonstrates why parents may demand children even if the monetary return is negative and the fact that fertility is likely to fall as infant and child mortality decrease.

Section 2 discusses children as insurance and alternative strategies. The two-period model with uncertain income in one period is presented and analysed in Sect. 3. Possible extension to a multi-period model is also discussed. Sect. 4 concludes and sums up.

2. Insurance, alternative strategies and children

Disruption of a household's income stream may result from disability or death of a person, who provides a significant labour input, as well as from adverse weather conditions, such as flooding or drought. Other causal factors are the risk of depredation and patriarchal risks,[2] which are primarily – but not exclusively – faced by the rural population in developing countries. Although households in developed countries and in parts of the urban areas in developing countries have ready access to insurance, either from private companies or through state-funded initiatives, poor households in the urban areas and most of the people living in the rural areas of developing countries do not.[3]

With absent or incomplete insurance markets, households need to rely on alternative income and consumption smoothing strategies. While it is unlikely that a household will rely solely on one strategy they are presented separately here. Five possibilities are considered: Saving, borrowing, public sector support, "traditional" systems of support and children.[4]

For saving or borrowing to be a viable means of insurance, a household needs a surplus in the other periods. If there is a surplus, accumulation can take place in cash, commodities, livestock or land. The first three are subject to depreciation of value, theft and costly storage, and since the duration of the adverse condition is normally unknown there is a risk of using up savings and borrowings before conditions improve. Hence, borrowing and saving can in many cases only provide relatively short-term relief.

Land can generate income, but yields vary with the weather. Furthermore, markets for land tend to be thin or nonexistent and sale of land leads to lower future income. Finally, land must be closely managed and acquiring large amounts of land means that the household must either be large or hire outside labour.[5] If a household relies on borrowing and uses land as collateral it faces extra hardship if it defaults on the loan since its earnings capacity will diminish. The household can also rely on the public sector, but public support varies from setting to setting and may be very unreliable or directly absent (Cain 1981).

The fourth strategy is to use the "traditional" systems of support. These include the village, the commons, and the extended family. Townsend (1994) examines whether the village as an institution can insure its inhabitants against bad weather conditions or other adverse conditions, but fails to find any strong support for the hypothesis. For both the village and the commons a high degree of co-variation in risks is likely, making it difficult to provide support when it is most needed. With respect to the extended family, Cain (1981) finds that a large part of the distress sale of land is to closely related kin, such as a brother. Since he could buy the land, it is also likely that he would have the money to help the relative in need, but decided not to. Rosenzweig and Stark (1989) find that daughters migrate to other villages to marry to mitigate their families' income risk.

The final possibility considered here is to use children as a substitute for insurance. Children can help either by working at home or as wage labour,[6] and older children who either have their own household or have migrated can make transfers to their parents.[7] It is important to note that even if wages are depressed, a household still gains from a large number of working children, provided that income covers costs. A child's consumption can also be reduced in case of adverse conditions, implying that the net return need not decrease much even with lower wages.[8] The most obvious reason why children with their own household or migrated family members want to remit money is family ties, also refered to as altruism or what Nugent (1985) calls loyalty. It follows that children are likely to be more reliable as a means for insurance than more distant family.[9] Finally, if the household is in dire straits the parents may actually "sell" their children as bounded labour. There is usually an underlying presumption that children should be of a certain age and in some societies of a specific sex to serve as a substitute for insurance. Nevertheless, the argument that only boys can act as insurance carries less weight if one accepts the point of Rosenzweig and Stark (1989) refered to above, because a larger number of girls leads to more connections with other households.

Children, when seen as a substitute for insurance, have three special properties. First, the expected net return of an additional child need not be positive for risk averse parents to have another child, since by definition they are willing to give up some of their income in order to reduce the risk. Hence, the insurance argument can contribute to understanding why studies, such as Cain (1982) and Lindert (1983), of the net return to children have failed to find any large positive return to children.

Secondly, children are a very general means of risk diversification and are not "used up" to the same extent as savings or borrowings. This means that children are in some aspects more like an annuity than an insurance policy. Both consumption and work effort of the children can change, however, making them closer to standard insurance. If parents derive utility from their children's consumption and education it is likely that if the family is well off the children will work less, consume more and possibly go to school. The parents can then increase the workload of the children and decrease the consumption as discussed above if needed.

Thirdly, children are only an incomplete substitute for insurance. They have a long maturing time, during which they are potentially very expensive, they may die before being able to provide any return to their parents, and there is no way *a priori* of knowing the sex or ability of the child. Furthermore, the number of children can take only discrete values. Hence, children are a crude substitute for insurance, but possibly better than the alternatives.

Three studies provide empirical support for the hypothesis that children provide general insurance against various risks. De Vany and Sanchez (1977) analyse the effect of land reform in rural Mexico and find that uncertain land tenure rights associated with the ejido system, in which land is granted to individual families on a usufruct basis and where land cannot be bought, sold, leased or mortgaged, leads to high fertility. They conclude that: "Children function as surrogate capital instruments, or securities, which permit parents to partially bridge the incompleteness of markets in claims to uncertain, future states" (De Vany and Sanchez 1977, p 761).

Cain (1990) analyses the relation between risk and fertility in two villages in Northern India. It is shown that although weather induced risk is relatively small and common property resources are available there are considerable "predatory" (political depredation) and patriarchal risks. This combined with semi-feudal social relations, which mean poor access to credit and little effect of state interventions, lead to a higher demand for sons compared with the villages in Southern India, studied in Cain (1981), where the risk environment is more benign and access to insurance substitutes easier.

Finally, Das Gupta (1995) examines fertility decline in the Ludhiana District, Punjab. Total fertility began to decline around 1940; well before the onset of family planning programmes and the initiation of the Green Revolution in 1966. According to Das Gupta this decline in fertility came about as a result of increased security against mortality peaks and food shortages. The improvement is partially due to the expansion of irrigation, which meant that "... both the *level* and the *variance* of yields were improved" (Das Gupta 1995, p 484).

3. The model

Consider a two-period decision problem for a household that faces a certain income in the first period and uncertainty about income and child survival in the second period. The household decides on the number of births in period one. In the second period the household's income is revealed together with the number of surviving children.

The number of births and the number of surviving children are assumed to be discrete variables. Let $N \in \{0, 1, 2, \dots\}$ denote the number of births and $n \in \{0, 1, \dots, N\}$ the number of surviving children in period two. It is assumed that the survival probability of each child is $s \in [0, 1]$, with s independent of the number of children and taken as given by the household. Hence, the probability that n children out of N births survive follows a binomial distribution with the density function

$$b(n, N, s) \equiv \binom{N}{n} s^n (1 - s)^{N-n}. \tag{1}$$

First period household income is given by y_1. In the second period there are two states of the world $x \in \{1, 2\}$, and household income is

$$y_2(x) = \begin{cases} \bar{y} & \text{if } x = 1 \\ \underline{y} & \text{if } x = 2 \end{cases}$$

The probability of state 1 is $p(1)$ and the probability of state 2 is $p(2) = 1 - p(1)$.

Each birth carries with it a constant cost h, so that the total cost of N births in the first period is hN. In the second period income minus expenditures for each surviving child is h. Hence, total income from n surviving children is hn. Since the cost and income factors are assumed to be equal, there can never be a pecuniary gain from having children even if they all survive. This corresponds to a stochastic rate of interest that is either zero or negative. If the second period income is known this implies that the household demands children only if the second period income is sufficiently lower than the first period income, assuming that the two period utility functions are identical.[10] It follows that if the expected second period income is equal to the first period income then any demand for children is due to the uncertainty of future income, again assuming that the two period utility functions are identical. It is in this sense that children serve as insurance.[11]

The choice of N determines consumption in period one as

$$c_1 = y_1 - hN. \tag{2}$$

The maximum number of births the household can have in the first period is $\left[\frac{y_1}{h}\right]$ or the biological maximum, which for simplicity is assumed to be higher than the budget constrained maximum. Consumption in the second period is the stochastic variable

$$c_2(x, n) = y_2(x) + hn. \tag{3}$$

The household is assumed to have a von Neumann-Morgenstern expected utility function

$$\bar{U}(c_1, c_2(x, n)) = \sum_{x,n} \bar{u}(c_1, c_2(x, n))p(x)b(n, N, s). \tag{4}$$

Assuming additive separability in consumption in the two periods, expected utility is

$$\tilde{U}(c_1, c_2(x, n)) = u^1(c_1) + \sum_x p(x) \sum_n b(n, N, s)u^2(c_2(x, n)). \tag{5}$$

Furthermore, both the first $u^1(c_1)$ and second period utility function $u^2(c_2(x, n))$, defined on sure amounts of consumption in each period, are assumed to be strictly increasing and concave in consumption. The household decides on the number of births rather than directly on consumption. Therefore, the expected utility of N births, for given s and p, is

$$U(N; s, p) = u^1(y_1 - hN) + \sum_x p(x) \sum_n b(n, N, s)u^2(y_2(x) + hn). \tag{6}$$

The household maximises (6) subject to the first period budget constraint (2).

3.1. *Deciding on the optimal number of births*

To analyse the optimal choice of the household one needs the discrete equivalent of the first and second order derivatives. The "marginal first period utility" of an additional birth is denoted by

$$u^1_N(y_1 - hN) \equiv u^1(y_1 - h(N + 1)) - u^1(y_1 - hN). \tag{7}$$

Since $u^1(c_1)$ is strictly increasing in c_1 and taking account of (2) this is negative. The change in the marginal first period utility due to an extra birth is given by

$$u^1_{NN}(y_1 - hN) \equiv u^1_N(y_1 - h(N + 1)) - u^1_N(y_1 - hN). \tag{8}$$

This is also negative due to the concavity assumption of the first period utility with respect to consumption. Since the first period utility is defined over the continuous variable c_1,

$$\begin{aligned}\int_{y_1-h(N+2)}^{y_1-h(N+1)} \frac{d}{d\mu}u^1(\mu)\,d\mu &= u^1(y_1 - h(N + 1)) - u^1(y_1 - h(N + 2)) \\ &= -u^1_N(y_1 - h(N + 1))\end{aligned}$$

and

$$\int_{y_1-h(N+1)}^{y_1-hN} \frac{d}{d\mu} u^1(\mu)\, d\mu = u^1(y_1 - hN) - u^1(y_1 - h(N+1))$$
$$= -u^1_N(y_1 - hN).$$

Together with (8) this leads to

$$u^1_{NN}(y_1 - hN) = -\int_{y_1-h(N+2)}^{y_1-h(N+1)} \frac{d}{d\mu} u^1(\mu)\, d\mu + \int_{y_1-h(N+1)}^{y_1-hN} \frac{d}{d\mu} u^1(\mu)\, d\mu.$$

By assumption $\frac{d}{dc_1} u^1(c_1)$ is non-increasing in c_1. Hence

$$u^1_{NN}(y_1 - hN) \leq 0.$$

That is, first period utility is decreasing and concave in the number of births for given first period income. Note that u^1_N and u^1_{NN} are defined only for $N \leq \left[\frac{y_1}{h}\right]$.

The marginal second period utility of an extra surviving child, given by

$$u^2_n(y_2(x) + hn) \equiv u^2(y_2(x) + h(n+1)) - u^2(y_2(x) + hn), \tag{9}$$

is always positive. This follows from the assumption that second period utility is strictly increasing in consumption. The change in marginal second period utility due to an extra surviving child,

$$u^2_{nn}(y_2(x) + hn) \equiv u^2_n(y_2(x) + h(n+1)) - u^2_n(y_2(x) + hn), \tag{10}$$

is negative or equal to zero following the same arguments as for the change in marginal first period utility.

With respect to the expected utility function define the marginal change in expected utility from an additional birth as

$$U_N(N; s, p) \equiv U(N+1; s, p) - U(N; s, p). \tag{11}$$

Let the change in this marginal expected utility due to one more birth be denoted as

$$U_{NN}(N; s, p) \equiv U_N(N+1; s, p) - U_N(N; s, p). \tag{12}$$

These two expressions are defined for $N \geq 0$ and $N \leq \left[\frac{y_1}{h}\right]$. The following two relationships emerge[12]

$$U_N(N;s,p) = u_N^1(y_1 - hN) + s\sum_x p(x)\sum_{n=0}^{N} b(n,N,s)u_n^2(y_2(x)+hn) \tag{13}$$

$$U_{NN}(N;s,p) = u_{NN}^1(y_1 - hN) + s^2\sum_x p(x)\sum_{n=0}^{N} b(n,N,s)u_{nn}^2(y_2(x)+hn) \leq 0. \tag{14}$$

The interpretation of (13) is close to the standard first order derivative in a maximisation problem. An additional birth leads to a cost in foregone first period utility, i.e. is the first part of (13). If this additional child does not survive to the second period there is no second period utility gain. If the child survives the household has one extra child in each of the possible outcomes of child survival and income states, captured by $\sum_x p(x)\sum_{n=0}^{N} b(n,N,s)u_n^2(\cdot)$. The assumption that the child has a survival probability of s leads to (13). Equation (14) shows that the expected utility function is concave.

Proposition 1. *There exists a solution to the household's maximisation problem and the optimal number of births is either a unique number or there are two neighbouring numbers that are both optimal.*

Proof. Since the number of births N is bounded from below by zero and from above by $\left[\frac{y_1}{h}\right]$ and can only take integer values there must be a finite number of possible choices. Hence a solution must exist.

Let $N(s,p)$ denote the largest optimal number of births given s and p. Owing to the concavity of $U(\cdot)$, it must satisfy the following conditions

$$U_N(N(s,p);s,p) < 0, \quad \text{and} \tag{15a}$$

$$U_N(N(s,p)-1;s,p) \geq 0. \tag{15b}$$

If

$$U_N(N(s,p);s,p) < 0, \quad \text{and}$$

$$U_N(N(s,p)-1;s,p) > 0,$$

it follows that $N(s,p)$ is the only optimal number of births given that $U_{NN}(\cdot) < 0$. If

$$U_N(N(s,p);s,p) < 0, \quad \text{and}$$

$$U_N(N(s,p)-1;s,p) = 0$$

both $N(s,p)$ and $N(s,p)-1$ are optimal. Finally, if

$$U_N(0; s, p) < 0$$

the optimal solution is $N(s, p) = 0$. ■

Having two optimal numbers of births might be seen as problematic. It is unlikely, however, that this situation arises given that a very small change in the parameter values would instead lead to a single optimal number of births. For brevity, it is assumed in the following proofs that the optimal solution is an interior one.

3.2. Risk aversion and changes in income

The optimal number of births depends on, among other variables, the household's present and future income and its degree of risk aversion. With respect to future income two effects are of interest here: The effect of a change in the level of income and the effect of a change in the dispersion of income.

Proposition 2. *The optimal number of births is non-increasing for increasing probability of higher second period income.*

Proof. Since $p(1)$ and $p(2)$ must sum to one, the marginal utility from an additional birth is

$$\begin{aligned} U_N(N; s, p) = u_N^1(y_1 - hN) \\ + s\Big[p\sum_{n=0}^{N} b(n, N, s)u_n^2(\bar{y} + hn) \\ + (1-p)\sum_{n=0}^{N} b(n, N, s)u_n^2(\underline{y} + hn)\Big]. \end{aligned} \tag{16}$$

Differentiating this expression with respect to p, which indicates the probability of high second period income, leads to

$$\frac{\partial}{\partial p} U_N(N; s, p) = s\sum_{n=0}^{N} b(n, N, s)[u_n^2(\bar{y} + hn) - u_n^2(\underline{y} + hn)]. \tag{17}$$

Since the second period utility is increasing and concave in consumption and $\bar{y}$ is larger than $\underline{y}$, the derivative must be negative for all N. That is,

$$\frac{\partial}{\partial p} U_N(N; s, p) < 0. \tag{18}$$

Evaluating this at $N = N(s, p)$ leads to

$$U_N(N(s, p); s, p') < U_N(N(s, p); s, p) \quad \text{for } p' > p. \tag{19}$$

Using the optimality condition (15a) yields

$$U_N(N(s,p); s, p') < 0. \tag{20}$$

Because of the concavity of $U(\cdot)$

$$U_N(N; s, p') < 0 \quad \text{for } N \geq N(s,p).$$

Furthermore,

$$U(N(s,p); s, p') > U(N(s,p) + 1; s, p') > \cdots$$

Hence, $N > N(s,p)$ cannot be optimal at p' and $N(s,p)$ must therefore be locally non-increasing in p. The global result is[13]

$$N(s, p') \leq N(s,p) \quad \text{for } p' > p. \qquad \blacksquare$$

Ruling out the case where changes in the level of expected income has no effect on the number of births the interpretation of Proposition 2 is that an increased probability of high future income leads to less demand for insurance and therefore fewer births. A similar effect can be shown to arise if the probability distribution remains the same, but either the income in the low income state, the income in the high income state or both are increased. The higher the expected future income, relative to the present income, the more willing the household is to take the risk of a low future income. Hence, there is less need for insurance. While an increase in the probability of high income or an increase in either low or high income may increase or decrease the variance of income, this effect is always dominated by the level effect, at least as long as the lower income is not decreased. Nevertheless, as indicated by the following proposition the dispersion of future income can also affect the demand for children.

Proposition 3. *A mean-preserving spread of future income cannot lead to a lower optimal number of births.*

Proof. By assumption the expected future income is

$$E[y_2] = p\bar{y} + (1-p)\underline{y}.$$

Let d be an arbitrary constant and let $\pi \in [0,1]$ be the probability that $\frac{d}{\pi}$ is added to second period income and $1-\pi$ the probability that $\frac{d}{1-\pi}$ is substracted from second period income in each state, then a mean-preserving spread of future income is

$$E[y_2] = p\left[\pi\left(\bar{y} + \frac{d}{\pi}\right) + (1-\pi)\left(\bar{y} - \frac{d}{1-\pi}\right)\right]$$
$$+ (1-p)\left[\pi\left(\underline{y} + \frac{d}{\pi}\right) + (1-\pi)\left(\underline{y} - \frac{d}{1-\pi}\right)\right].$$

Let $\tilde{U}_N(N; s, p)$ denote the marginal change in expected utility from an additional birth when the distribution of expected income is the mean-preserving spread. Then

$$\begin{aligned}
&\tilde{U}_N(N; s, p) - U_N(N; s, p) \\
&\quad = s\Bigg\{ p \sum b(n, N, s) \left[\pi u_n^2\left(\bar{y} + \frac{d}{\pi} + hn\right) \right. \\
&\qquad \left. + (1-\pi)u_n^2\left(\bar{y} - \frac{d}{1-\pi} + hn\right) - u_n^2(\bar{y} + hn) \right] \\
&\qquad + (1-p) \sum b(n, N, s) \left[\pi u_n^2\left(\underline{y} + \frac{d}{\pi} + hn\right) \right. \\
&\qquad \left. + (1-\pi)u_n^2\left(\underline{y} - \frac{d}{1-\pi} + hn\right) - u_n^2(\underline{y} + hn) \right] \Bigg\} \geq 0.
\end{aligned}$$

The inequality follows from the assumption that u^2 is increasing and concave. Since the marginal expected utility from an additional birth is higher for the mean-preserving spread distribution than for the original distribution it follows from Proposition 1 that it cannot be optimal to have fewer births. ∎

Proposition 3 demonstrates that a mean-preserving spread in future income cannot lead to a lower demand for children. Clearly, the result would be the same if the high income is increased and the low income is decreased, keeping the mean constant. The proof also indicates that the more risk averse the household is (i.e. the more concave the second period utility function is), the higher is the likelihood that the optimal number of births will increase.

It is likely that adverse conditions in developing countries can lead to a future income so low that it threatens the very survival of the household. The effect of this possibility on the demand for children depends on the characteristics of the utility function as consumption approaches zero. Assuming that the marginal utility goes to infinity as future consumption goes to zero it would appear that the household would demand an infinite number of children or in the real world have as many children as biologically possible. The maximum number of births is, however, also constrained by the first period budget constraint, so the marginal utility of consumption in the first period would also increase substantially as N approaches $\left[\frac{y_1}{h}\right]$. Proposition 2 still holds but it is less likely that an increase in the high income would generate any observable effect on the observed number of births.

The implication is that even families who are relatively richer in the sense that their high second period income is higher than others would tend to have a large number of children if they faced a risk of zero or very low income in some periods. This would seem to support the conclusion by Cain (1986) that in rural Bangladesh, where the important sources of risk are endemic, "... one should not expect fertility to vary systematically across region or economic status". If everybody experiences a high risk of a very low income no matter their status there would not be much difference between fertility levels due to security considerations.

Families differ not only with respect to their expected income but also with respect to their present income. The model can also be used to analyse the impact of present income on fertility. This is done in Propostion 4.

Proposition 4. *For a given expectation of second period income the optimal number of births cannot be higher for a lower first period income than for a higher first period income.*

Proof. Use (13) to find

$$U_N(N;\bar{y}_1) - U_N(N;\underline{y}_1)$$
$$= u_N^1(\bar{y}_1 - hN) - u_N^1(\underline{y}_1 - hN).$$

This is positive because of the concavity assumption and therefore

$$U_N(N;\bar{y}_1) > U_N(N;\underline{y}_1). \tag{21}$$

From the optimality condition

$$U_N(N(\bar{y}_1);\bar{y}_1) < 0.$$

Using this and (21) yields

$$U_N(N(\bar{y}_1);\underline{y}_1) < U_N(N(\bar{y}_1);\bar{y}_1).$$

This leads, together with the concavity assumption, to

$$U_N(N;\underline{y}_1) < 0 \quad \text{for } N \geq N(\bar{y}_1).$$

Thus, it can never be optimal to have more children when the present income is lower than the number one would have if income were higher. ■

Ruling out the uninteresting case where first period income has no effect on N, the optimal number of births is lower if the present income is lower. Mostly, in empirical analyses of the demand for children, only present income or some proxy for income is observed together with the number of children. According to Proposition 4 there should be a positive relation between income and fertility in a given period, but Proposition 2 predicts a negative relation between future expected income and the number of births. Hence, to determine the demand for children it is not sufficient to observe present income, one also needs people's own assesment of future expected income and its variability.

In poverty striken environments with little possibility for investment in children and assuming that the variation in expected future income across households is not too big there will be a positive relationship between present income and fertility according to both the theory presented here and the standard beckerian model. In the insurance model the reason for this is that since everybody has approximately the same distribution of expected income only proposition 4 will be in effect and any difference in fertility must then be a result of differences in present (or past) income. For the beckerian model fertility will be positively correlated with income if the quality of the children is assumed to be basically constant. Hence, simply regressing fertility against present (or some measure of past) income may lead to accepting a hypothesis of children as a standard consumer good. Thus, one concludes that better-off people will tend to have more children. Yet, the true underlying hypothesis may be that children serve as a substitute for insurance. The result is that as people's expected future income increase their fertility does in fact decline.

The above argument assumes as stated that the variation in expected income across the households in the sample is not too large. The other extreme is that there is little variation in present income but variation in expected income. In this case a family with a high expected future income will have fewer children than a family with low expected future income, even if their present income is the same. Therefore, when analysing the demand for children it is sensible to control for expected future income. This variable is difficult to observe. Thus, a proxy, such as education, needs to be found.

Education is a possible proxy for three reasons. First, it is typically correct that the more educated a person is, the higher the expected future income.[14] Secondly, more education is likely to lead to less variation in expected future income, even if it does not substantially increase expected income. Finally, since education provides people with the ability to collect and process information, they are in a better position to asses their future income and to take steps to prevent a very low income state occuring. If education is a good proxy for future income and its variation this model could help explain part of the inverse relationship between fertility and education. Empirical studies of the relation between education and fertility have found that the mother's education has the strongest impact on household fertility (Schultz 1997). This corresponds with the prediction of this model if children are primarily demanded as insurance by women as argued by Cain (1982) and Nugent (1985).

Furthermore, various studies have shown a negative relation between infant and child mortality and the education of mothers.[15] Hence, education has two effects that both tend to lower the number of births. First, higher education means less need for insurance because of higher expected income and lower variation in income. Secondly, the household needs fewer birth since child mortality decreases with education.

The opposite effects arising from an increase in present income and an increase in future income are especially important if the household does not fully realise these changes when they take place. Consider an exogenous increase in the probability of high income in future periods that is not fully realised by the household when it occurs. The result is a higher (expected) fertility, since the household is now likely to have more periods with high income, while they act as if they have a lower expected income during the updating of their beliefs. Taking account of this learning effect may explain part of the experienced increase in fertility in the beginning of a country's development. Clearly the higher fertility is sub-optimal for the household. There is therefore a potential welfare gain if the household knew the true distribution.

3.3. Effects of changes in survival probability

Beside the level and variance of income discussed above, the survival probability of the children is also important in determining the optimal number of births, since it influences the return on children. The following proposition examines the effect on fertility.

Proposition 5. *For a sufficiently risk averse household, the optimal number of births is first non-decreasing and then non-increasing in the survival probability.*

Proof. From the proof of Proposition 2, a sufficient condition for the optimal number of births to be non-increasing in the survival probability is that the marginal utility of an additional birth is decreasing in the survival probability, when evaluated at the optimal number of births. Differentiating $U_N(\cdot)$ with respect to s leads to[16]

$$\frac{\partial}{\partial s} U_N(N; s, p) = \sum_x p(x) \Bigg[\sum_{n=0}^{N-1} \{sNb(n, N-1, s) - B(n, N, s)\} \times u_{nn}^2(y_2(x) + hn) + u_n^2(y_2(x) + hN) \Bigg]. \tag{22}$$

The derivative of the marginal utility of an additional birth with respect to the survival probability depends on the sign of the expression inside the curled brackets and the curvature of the second period utility function. There are three possible cases. First, for low s, and N not too large, the expression inside the brackets is negative for all n. Since the second period utility is concave by assumption, an increase in the survival probability will unambigously lead to a positive value of the derivative. Hence, the optimal number of children must be non-decreasing in the survival probability for low values of the survival probability. Secondly, for medium range values of s, the expression can be both positive and negative. For low values of n the expression will be positive and for higher values it will be negative. Therefore, the derivative can be both positive and negative, depending on the concavity of the second period utility function. A negative value is more likely the more risk averse the household is and the more positive the third order derivative is. Finally, for higher values of s, the expression is positive for all n.[17] Again, the more risk averse the household is the more likely it is that the derivative is negative and therefore that the optimal number of births is non-increasing in the survival probability.[18] ■

An increase in the survival probability of children has two effects on the demand for children as insurance. First, increased survival probability is equivalent to a higher return to births (less wasted resources). Secondly, the higher expected number of survivers leads to a higher expected consumption in the future. While the substitution effect tends to raise the optimal number of births the income effect has the opposite effect. If the income effect dominates the optimal number of births will decrease. Proposition 5 states that the more risk averse a household is the more likely it is that the income effect will at one point dominate the substitution effect when the survival probability is increased. Hence, the model is able to illustrate the observed fall in fertility following a decline in infant and child mortality, provided households are indeed risk averse.

3.4. Extension to a multi-period model

The household may have some overall idea about the number of surviving children it wants, but the decisions on timing and number of births are influenced by present income and the number of surviving children. It follows that the household's fertility choice is potentially better described as a stochastic dynamic programme. As shown in Pörtner (1998) the results for the two-period model do carry over into the three-period model, where the parents can have children in the first two periods and where income is uncertain for the last two periods.[19]

Besides the propositions equivalent to the ones in the two-period model, one can show that the optimal number of second period births is non-increasing in the number of surviving children from the first period. There are two effects from an extra surviving child in the second period. First, the child will increase the expected income in both the low and the high income states. This would tend to reduce the demand for children in the second period. Secondly, with the additional child the household has a higher present income, which implies a higher demand for children. When the utility functions for period two and three are identical the first effects dominates the second.

Finally, it is possible to show that the optimal number of births in the first period is non-increasing in both the survival rate of the second period births and in the probability of a high third period income. This assumes that a unit decrease in the number of surviving children from the first period does not lead to more than a unit increase in the optimal number of births in the second period. These results are the natural extensions of the propositions dealing with the effects of changes in expected income and survival in the next period. They indicate that a change in expectations will have the same qualitative effect whether the change concerns the next period or one of the following periods.

4. Conclusion

The main hypothesis of this paper is that children can act as a security asset when insurance and credit markets are either absent or poorly functioning. Incorporating this into a model with uncertain future income and child survival and a discrete representation of the number of children three main results emerge. First, for risk-averse households, the number of births is decreasing in the survival probability for realistic values of the survival probability. Secondly, a higher expected future income leads to a lower number of births. Thirdly, for a given expectation of future income the number of births in a period is increasing in income. Together, the second and third result imply that standard empirical analysis of the demand for children where the number of children is regressed against income will not be able to capture the entire effect. Two households with similar present income may have different expected income and hence different number of children.

The simple model put forward here provides a different perspective on a variety of stylised facts with respect to fertility in developing countries. First, the model indicates why a negative return to children does not necessarily imply that parents do not demand children. Secondly, the observed positive relation between income and fertility at low income levels, and the negative relation between fertility and income at higher income levels can both be accomodated within the model. Thirdly, parallel with the results on income, the model provides a possible reason for the negative relation between fertility and education, without resorting to the assumption that children are necessarily very time intensive for the mother, something which may not be true in many developing countries.[20] Finally, the model can also help explain the decline in fertility when infant and child mortality decrease.

With respect to the policy implications of the above results, any policy that could reduce the variability of income or raise the level of income or both would be beneficial to the households and reduce fertility. One way of achieving this could be irrigation as discussed in Das Gupta (1995). Better functioning markets, both for credit, insurance and goods would also raise the utility of the household and reduce fertility. The same would be the case with reliable state intervention in times of crisis, more secure property rights and access to an impartial and well-functioning judicial system. Better health care will also improve the situation of the households by reducing the risk of incapacitating illnesses and lowering infant and child mortality. The reduced risk of illness would reduce the need for insurance and hence children, and at the same time improve the quality of life for the parents. Policies that create more opportunities for women, especially by raising their earnings capacity, would also lead to lower fertility and improved welfare. One such policy that is likely to be effective is providing education to women.

Future work on this topic could progress along both theoretical and empirical lines. On the theoretical side an extension of the model to include endogenous mortality risk or other forms of investments in child "quality" could be of interest. On the empirical there is so far very little direct evidence on whether children serve as a substitute for insurance. This highlights the need for formulating further testable results and collecting the needed data. At the moment there is at least one result that is testable with data. An implication of Proposition 3 is that household with less expected variation in their income would have lower fertility for a given income. Hence, one direct test of the model presented here is to analyse the effects of a mean-preserving spread of future income on fertility. This can be done using, for example, a combination of household and agricultural data.

Endnotes

1. See Nugent (1985), Nugent and Anker (1990), Ehrlich and Lui (1991, 1997, 1998), Schultz (1997) and references herein.
2. Patriarchal risk is the special risk faced by women and includes risk of widowhood, divorce or abandonment. The risk of depredation is the risk of a direct threat to one's property, either through fraud or expropriation. Cain (1981) consideres the different types of risks in more detail.
3. Nugent (1985) discusses why insurance companies find it unprofitable to operate in these areas.
4. Cain (1981) and Nugent (1985) examine these strategies and their effectiveness in more detail.
5. Cain (1983, 1985) examines the problems associated with investing in land for security purposes.
6. In many less developed countries children from poor families begin to work a substantial number of hours per day from age 5–6 (Cain 1977, 1982; Dasgupta 1993, p 359).

[7] Alternatively, a parent can migrate while the children stay behind. With respect to migration the hypothesis of this paper can be seen as an extension of Stark's (1991, ch. 4 and 5) suggestion that migration is in part a response to risk aversion.
[8] Empirical evidence on the effects of adverse conditions on the consumption of children is not conclusive. The intra-household distribution of food varies between different locations as well as with the severity of the situation (Rosenzweig and Schultz 1982, Harriss 1990, Dasgupta 1993).
[9] For further discussion of why migrants remit, such as altruism and self-enforcing contracts, see Lucas and Stark (1985), Stark (1991, ch. 15), Cox and Stark (1996) and Lillard and Willis (1997).
[10] There would also be a demand for children if the income factor was sufficiently higher than the cost factor or if the utility function allowed for direct utility of children.
[11] This is also known as precautionary saving, which is defined "... as the extra saving caused by future income being random ..." (Leland 1968, p 465). See also Kimball (1990).
[12] The details of the derivations are available from the author on request.
[13] The details of this proof are available from the author.
[14] Foster and Rosenzweig (1996) analyse the return to schooling during the green revolution in India. See also the discussion of the effects of schooling in Rosenzweig (1995).
[15] Examples are Bhuiya and Streatfield (1991) and Sandiford, Cassel, Montenegro, and Sanchez (1995).
[16] The derivation is available from the author on request.
[17] Sah (1989, p 23) finds that the expression is positive if $s \geq 0.81$ and $N \leq 12$.
[18] Using simulation it can be shown that for $y_1 = 500, \bar{y} = 900, \underline{y} = 100$, $h = 25, p = 0.5$, and utility functions $u^i(\cdot) = \frac{c_i^{1-\gamma}}{1-\gamma}, i = 1, 2, \frac{\partial}{\partial s} U_N(N; s, p) < 0$ for $\gamma = 1.67$ and s close to one. Furthermore, the optimal number of births is decreasing in s for $\gamma = 2.14$.
[19] This also assumes that there cannot be a pecuniary return to having children. Furthermore, it is assumed that children are only at risk in their first period of life.
[20] Reasons for this include economies of scale in child-rearing with older siblings taking care of younger ones; the fact that many women work at home or at the family farm, which means that child-rearing and work is not exclusive activities; and that older family members often live with their children and hence can participate in the child-rearing if the mother works outside the home.

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